विघ्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्‌गुरु श्री रणछोड़दासजी महाराज

Subject : PHYSICS

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Q.1$_{ud/13}$ A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\frac{dx}{dt} = c = \frac{dy}{dt}$. Where a, b and c are constants. The acceleration of the particle is

(A*) $(6ac^2x + 2bc^2)\hat{k}$ (B) $(2ax^2 + 6by^2)\hat{k}$ (C) $(4bc^2x + 6ac^2)\hat{k}$ (D) $(bc^2x + 2by)\hat{k}$

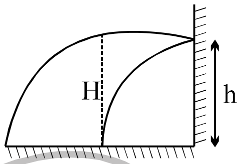
[Sol. $z = ax^3 + by^2$, $\frac{dz}{dt} = 3ax^2\frac{dx}{dt} + 2by\frac{dy}{dt}$

$$\frac{dz}{dt} = 3acx^2 + 2bcy, \quad \frac{d^2z}{dt^2} = 6acx\frac{dx}{dt} + 2bc\frac{dy}{dt}$$

$$\frac{d^2z}{dt^2} = (6ac^2x + 2bc^2)$$

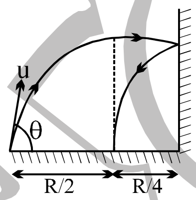
$$\vec{a} = (6ac^2x + 2bc^2)\hat{k}$$]

Q.2$_{kin/11/12/13}$ A stone is projected from a horizontal plane. It attains maximum height 'H' & strikes a stationary smooth wall & falls on the ground vertically below the maximum height. Assume the collision to be elastic the height of the point on the wall where ball will strike is:



(A) H/2 (B) H/4
(C*) 3H/4 (D) none of these

[Sol. Because horizontal velocity is constant so



$$T = \frac{2u\sin\theta}{g}$$

given $H = \frac{u^2\sin^2\theta}{2g}$, $u\sin\theta = \sqrt{2gH}$

$T = \frac{2\sqrt{2gH}}{g}$ at the time of hitting the wall

The horizontal distance covered is $\frac{3R}{4}$, so time taken to cover horizontal distance $\frac{3R}{4}$

$$T' = \frac{3T}{4} = 3\sqrt{\frac{H}{2g}}, \quad h = \sqrt{2gH} \times 3\sqrt{\frac{H}{2g}} - \frac{1}{2}\times g\times a\times\frac{H}{2g}$$

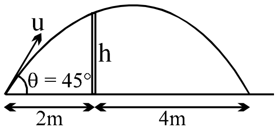
$$h = \frac{3H}{4}$$]

Q.3$_{kin/12/13}$ A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4 m from it. The height of the wall is

(A) 3/4 m (B) 2/3 m (C) 4/3 (D) 1/3 m

[Sol. $R = 6 = \frac{u^2\sin 2\theta}{g}$, $\theta = 45°$

$60 = u^2$, $u = \sqrt{60}$



$u\cos\theta = \sqrt{30}$, $u\sin\theta = \sqrt{30}$

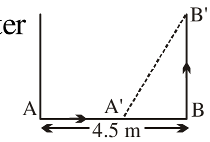
$t = \dfrac{2}{\sqrt{30}}$,

$h = \sqrt{30} \times \dfrac{2}{\sqrt{30}} - \dfrac{1}{2} \times 10 \times \dfrac{4}{30}$

$h = 2 - \dfrac{2}{3} = \dfrac{4}{3}$ m,

$h = \dfrac{4}{3}$ m]

Q.4$_{kin/13}$ Two particles instantaneously at A & B respectively 4.5 meters apart are moving with uniform velocities as shown in the figure. The former towards B at 1.5 m/sec and the latter perpendicular to AB at 1.125 m/sec. The instant when they are nearest is:

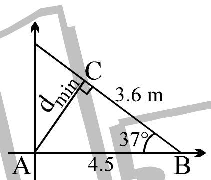


(A) 2 sec (B) 3 sec (C) 4 sec (D*) $1\dfrac{23}{25}$ sec

[Sol. $\vec{V}_1 = 1.5\hat{i}$, $\vec{V}_2 = 1.125\hat{j}$

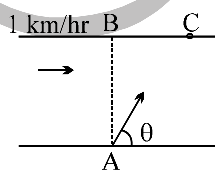
$\vec{V}_{21} = 1.125\hat{j} - 1.5\hat{i}$

$|\vec{V}_{21}| = \sqrt{(1.125)^2 + (1.5)^2} = 1.875$ m/s



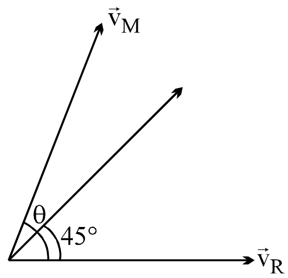
$t = \dfrac{3.6}{1.875} = 1.92 \text{ sec} = 1\dfrac{23}{25}$]

Q.5$_{kin/13}$ A river is flowing with a speed of 1 km/hr. A swimmer wants to go to point 'C' starting from 'A'. He swims with a speed of 5 km/hr, at an angle θ w.r.t. the river. If AB = BC = 400 m. Then the value of θ is:



(A) 37º (B) 30º (C*) 53º (D) 45º

[Sol. Condition for reaching the point C



$\tan 45° = \dfrac{v_y}{v_x}$, $v_y = v_x$

$(v_R + v_M\cos\theta) = v_M\sin\theta$

$1 + 5\cos\theta = 5\sin\theta$

$1 + 5\cos\theta = 5\sqrt{1-\cos^2\theta}$

On squaring,

$1 + 25\cos^2\theta + 10\cos\theta = 25 - 25\cos^2\theta$

$50\cos^2\theta + 10\cos\theta - 24 = 0$

On solving,

$\Rightarrow \theta = 53°$]

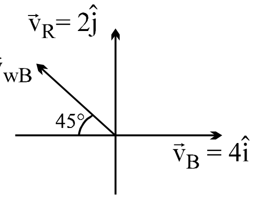
Q.6$_{kin/13}$ A boat is moving towards east with velocity 4 m/s with respect to still water and river is flowing towards north with velocity 2 m/s and the wind is blowing towards north with velocity 6 m/s. The direction of the flag blown over by the wind hoisted on the boat is:

(A*) north-west (B) south-east (C) $\tan^{-1}(1/2)$ with east (D) north

[Sol. $\vec{v}_{Bg} = \vec{v}_{BR} + \vec{v}_{Rg} = 4\hat{i} + 2\hat{j}$

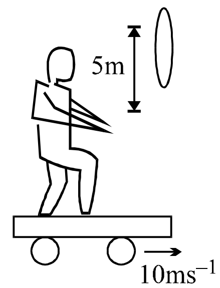
$\vec{v}_{wg} = 6\hat{i}$

$\vec{v}_{wB} = \vec{v}_w - \vec{v}_B = 6\hat{i} - 4\hat{i} - 2\hat{j} = 2\hat{i} - 2\hat{j}$



Direction will be north-west]

Q.7$_{kin/13}$ A girl is riding on a flat car travelling with a constant velocity 10 ms^{-1} as shown in the fig. She wishes to throw a ball through a stationary hoop in such a manner that the ball will move horizontally as it passes through the hoop. She throws the ball with an initial speed $\sqrt{136}$ ms^{-1} with respect to car. The horizontal distance in front of the hoop at which ball has to be thrown is



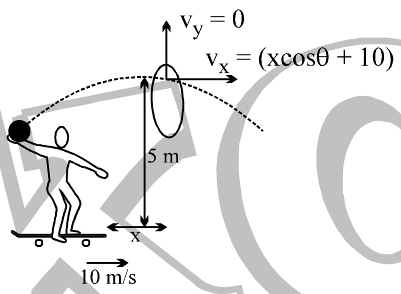
(A) 1m (B) 2m (C) 4m (D*) 16m

[Sol. $x = (10 + \sqrt{136}\cos\theta)t$ ...(1)

$v_y^2 = 0 = 136\sin^2\theta - 2 \times 10 \times 5$ ...(2)

$\sin\theta = \dfrac{5}{\sqrt{34}}, \cos\theta = \dfrac{3}{\sqrt{34}}$

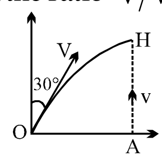
$5 = \sqrt{136} \times \dfrac{5}{\sqrt{34}} t - 5t^2$ ...(3)



$t^2 - 2t + 1 = 0 \Rightarrow t = 1$

So, x = 16 m]

Q.8$_{kin/13}$ A particle is projected with a speed V from a point O making an angle of 30° with the vertical. At the same instant, a second particle is thrown vertically upward from a point A with speed v. The two particle reach H, the highest point on the parabolic path of the first particle simultaneously, then the ratio V/v



(A) $3\sqrt{2}$ (B) $2\sqrt{3}$ (C*) $\dfrac{2}{\sqrt{3}}$ (D) $\dfrac{\sqrt{3}}{2}$

[Sol. $H = \dfrac{V^2\sin^2 60°}{2g}$, $\sqrt{\dfrac{2gH \times 4}{3}} = V = \sqrt{\dfrac{8gH}{3}}$

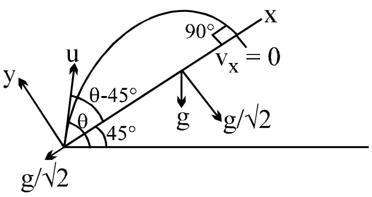
$v = \sqrt{2gH}$, $\dfrac{V}{v} = \dfrac{\sqrt{8gH/3}}{\sqrt{2gH}} = \dfrac{2}{\sqrt{3}}$

$\dfrac{V}{v} = \dfrac{2}{\sqrt{3}}$]

Q.9$_{kin/13}$ A particle is projected with a certain velocity at an angle θ above the horizontal from the foot of a given plane inclined at an angle of 45° to the horizontal. If the particle strike the plane normally then θ equals

(A) $\tan^{-1}(1/3)$ (B) $\tan^{-1}(1/2)$ (C) $\tan^{-1}(1/\sqrt{2})$ (D*) $\tan^{-1} 3$

[Sol. $v_x = u_x - \frac{g}{\sqrt{2}}t$



$$0 = u\cos(\theta - 45°) - \frac{g}{\sqrt{2}}t$$

$$t = \frac{\sqrt{2}u\cos(\theta - 45°)}{g}$$

$$y = u_y t - \frac{1}{2}\frac{g}{\sqrt{2}}t^2, \; y = 0$$

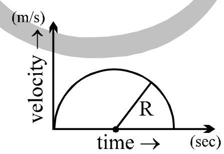
$$\frac{2\sqrt{2}u_y}{g} = t = \frac{2\sqrt{2}u\sin(\theta - 45°)}{g}$$

$$\frac{\sqrt{2}u\cos(\theta - 45°)}{g} = \frac{2\sqrt{2}u\sin(\theta - 45°)}{g}$$

$$\frac{1}{2} = \tan(\theta - 45°)$$

$$\frac{\tan\theta - 1}{1 + \tan\theta} = \frac{1}{2} \quad \Rightarrow \quad \tan\theta = 3 \quad \Rightarrow \quad \theta = \tan^{-1}(3)]$$

Q.10$_{kin/13}$ Velocity time graph of a particle is in shape of a semicircle of radius R as shown in figure. Its average acceleration from T = 0 to T = R is:



(A) 0 m/s^2 (B*) 1 m/s^2

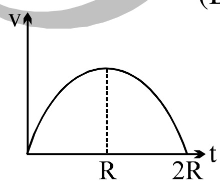
(C) R m/s^2 (D) 2R m/sec^2

[Sol. u = 0, V = R (at T = R)

T = R, V = u + at

R = 0 + a × R

a = 1 m/s^2]



Q.11$_{kin/13}$ A car is moving with uniform acceleration along a straight line between two stops X and Y. Its speed at X and Y are 2m/s and 14m/s. Then

(A) Its speed at mid point of XY is 15m/s

(B) Its speed at a point A such that XA : AY = 1 : 3 is 5m/s

(C*) The time to go form X to the mixed point of XY is double of that to go from mid point to Y.

(D) The distance travel in first half of the total time is half of the distance travelled in the second half of the time.

[Sol. (A) Not possible if acceleration is const.

(B) Velocity at mid-point $\quad v^2 = u + 2 \times a \times \frac{\ell}{4}$

$$106 = v^2 + 2 \times a \times \frac{3\ell}{4}$$

$$a = \frac{96}{\ell},\ v = \sqrt{52}$$

(C) Possible
(D) Not possible]

Q.12$_{kin}$ A particle having a velocity $v = v_0$ at $t = 0$ is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.

(A*) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$
(B) The particle will come to rest at infinity.
(C) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$.
(D*) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$.

[Sol. (A) $a = -\alpha\sqrt{v}$

$$\frac{dv}{dt} = -\alpha v^{1/2}$$

$$\int_{v_0}^{0}\frac{dv}{v^{1/2}} = \int_0^t -\alpha dt$$

$$\left[2v^{1/2}\right]_{v_0}^{0} = -\alpha t$$

$$2[-v_0^{1/2}] = -\alpha t$$

$$t = \frac{2\sqrt{v_0}}{\alpha}$$

(D) Velocity at any time t is

$$\int_{v_0}^{v}\frac{dv}{v^{1/2}} = -\int_0^t \alpha\, dt$$

$$[2v^{1/2}]_{v_0}^{v} = -\alpha t$$

$$2[v^{1/2} - v_0^{1/2}] = -\alpha t$$

$$v = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$$

$$v = \frac{d\alpha}{dt} = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$$

$$\int_0^{\alpha} d\alpha = \int_0^{\alpha}\left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2 dt$$

$$x = v_0 t + \frac{\alpha t^3}{12} - \frac{\alpha t^2 \sqrt{v_0}}{2}$$

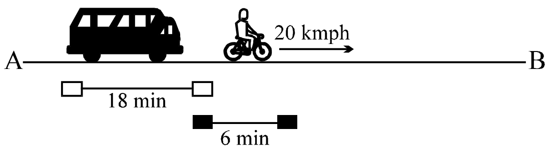
at $t = \dfrac{2\sqrt{v_0}}{\alpha}$

$$x = \frac{2}{3}\frac{v_0^{3/2}}{\alpha}$$]

Q.13$_{kin/13}$ Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with speed of 20km/h in the direction A to B, notices that a bus goes past him every t_1 = 18 minutes in the direction of motion, and every t_2 = 6 minutes in the opposite direction. What is the period T of the bus service? Assume that velocity of cyclist is less than velocity of bus

(A) 4.5 minutes (B) 24 minutes (C*) 9 minutes (D) 12 minutes

[Sol.



$$(v - 20)\ \frac{18}{60} = d = vt$$

$$(v + 20)\ \frac{6}{60} = d = vt$$

$3v - 60 = v + 20$

$v = 40$ kmph

$$(40 + 20) \times \frac{6}{60} = 40 \times T$$

$6 = 40\ T \quad \Rightarrow \quad T = 6/40$ hr $= 9$ min]

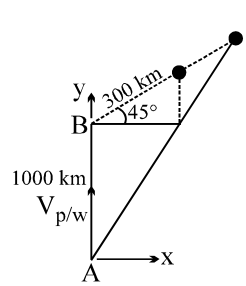
Q.14$_{kin/13}$ An airplane pilot wants to fly from city A to city B which is 1000 km due north of city A. The speed of the plane in still air is 500 km/hr. The pilot neglects the effect of the wind and directs his plane due north and 2 hours later find himself 300km due north-east of city B. The wind velocity is

(A*) 150km/hr at 45°N of E (B) 106km/hr at 45°N of E
(C) 150 km/he at 45°N of W (D) 106 km/hr at 45°N of W

[Sol. $V_{p/w} = 500 \text{ kmph } \hat{j}$

$V_{w/g} = v_x\hat{i} + v_y\hat{j}$

$V_{p/g} = v_x\hat{i} + (v_y + 500)\hat{j}$ kmph



In two hours

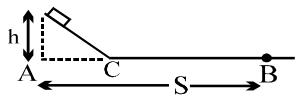
$S_{p/g} = 150\sqrt{2}\,\hat{i} + 1150\sqrt{2}\,\hat{j}$ km $= 2v_x\,\hat{i} + (2v_y + 1000)\,\hat{j}$

$\Rightarrow \quad v_x = 75\sqrt{2}$

$\quad\quad v_y = 75\sqrt{2}$

$V_{w/g} = 75\sqrt{2}\,\hat{i} + 75\sqrt{2}\,\hat{j} = 150$ kmph at 45° N of E]

Q.15$_{nlf/11/12/13}$ A block slides down on an icy hill of height h (as shown in figure) and stops after covering a distance CB. The distance AB is equal to 'S'. The coefficient of friction μ between the block & ice surface (inclined & horizontal) is



(A*) $\mu = \frac{h}{S}$ (B) $\mu = \frac{h}{\sqrt{S^2 - h^2}}$ (C) $\mu = \frac{S}{h}$ (D) data's are insufficient

[Sol. $W_G + W_f = \Delta KE$

$W_f = -W_G$

$-\mu mg\cos\theta \times \frac{h}{\sin\theta} - \mu mg(S - h\cot\theta) = -mgh$

$-\mu mgh\cot\theta - \mu mg(S - h\cot\theta) = -mgh$

$-\mu mS = -mgh$

$\mu = h/S$]

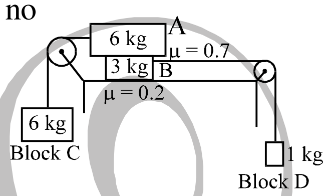
Q.16$_{nl/11/12/13}$ An arrangement of the masses and pulleys is shown in the figure. Strings connecting masses A and B with pulleys are horizontal and all pulleys and strings are light. Friction coefficient between the surface and the block B is 0.2 and between blocks A and B is 0.7. The system is released from rest (use $g = 10\ m/s^2$).

(*A) The magnitude of acceleration of the system is $2\ m/s^2$ and there is no slipping between block A and block B

(B) The magnitude of friction force between block A and block B is 42 N

(C) Acceleration of block C is $1\ m/s^2$ downwards

(D*) Tension in the string connecting block B and block D is 12 N



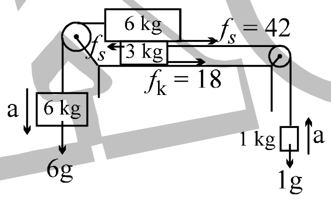
[Sol. (A) $6g - 18 - 1g = 16a$

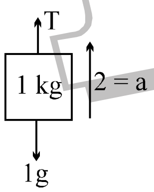
$a = 2\ m/s^2$ C moving down

(B) $f_s - 18 - 10 = 4 \times 2$

$f_s = 36$

(C) $a_c = 2\ m/s^2$ downward



(D) 

$T - 10 = 1 \times 2 \Rightarrow T = 12\ N$]

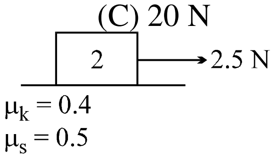
Q.17$_{nl/12/13}$ A body of mass 2 kg is placed on a horizontal surface having kinetic friction 0.4 and static friction 0.5. If the force applied on the body is 2.5 N, the frictional force acting on the body will be ($g = 10\ m/s^2$)

(A) 8 N (B) 10 N (C) 20 N (D) 2.5 N

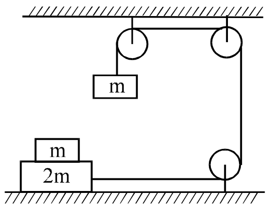
[Sol. $f_{l\cdot s} = \mu_s mg = 0.5 \times 2g = 10\ N$

Block is stationary $P < f_{l\cdot s}$

$\Rightarrow$ friction force $f = P = 2.5\ N$]



Q.18$_{nl/13}$ In the arrangement shown in figure, there is friction between the blocks of masses m and 2m which are in contact. The ground is smooth. The mass of the suspended block is m. The block of mass m which is kept on mass 2m is stationary with respect to block of mass 2 m. The force of friction between m and 2m is (pulleys and strings are light and frictionless) :



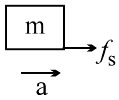
smooth ground

(A) $\frac{mg}{2}$ (B) $\frac{mg}{\sqrt{2}}$ (C*) $\frac{mg}{4}$ (D) $\frac{mg}{3}$

[Sol. mg = 4ma

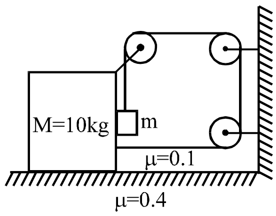
a = g/4

$f_s = ma = mg/4$



]

Q.19$_{nl/13}$ The maximum value of m(in kg) so that the arrangement shown in the figure is in equilibrium is given by



(A) 2 (B*) 2.5 (C) 3 (D) 3.5

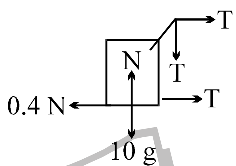
[Sol. Bigger block is not moving

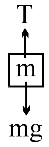
T = mg ...(1)

2T = 0.4 N ...(2)

T + 10 g = N ...(3)

String





$$\frac{2T}{T+100} = 0.4$$

1.6 T = 40

T = 25

$\Rightarrow$ m = 2.5 kg]

Q.20$_{nl/13}$ Two blocks, A and B, of same masses, are resting in equilibrium on an inclined plane having inclination with horizontal = α (>0). The blocks are touching each other with block B higher than A. Coefficient of static friction of A with incline = 1.2 and of B = 0.8. If motion is not imminent,

(A) $\alpha < 30°$ (B*) $(\text{Friction})_A > (\text{Friction})_B$

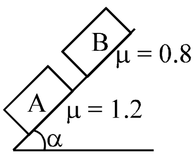
(C*) $\alpha < 45°$ (D) $(\text{Friction})_A = (\text{Friction})_B$

[Sol.

$(f_A + f_B)_{max}$

$2mg\sin\alpha < 2mg\cos\alpha$

$\tan\alpha < 1 \quad \Rightarrow \quad \alpha < 45°$



$f_A = mg\sin\alpha + N$

$f_B = mg\sin\alpha - N$

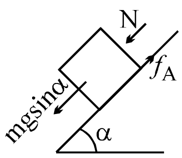
To show $N \neq 0$

For A

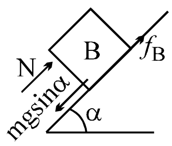
$$f_{b_{max}} = 0.8\, mg\cos\alpha$$

$\Rightarrow$ $N = mg[\sin\alpha - 0.8\cos\alpha]$

$N = mg\sec\alpha[\tan\alpha - 0.8]$



FBD of A



For $\alpha > \tan^{-1}(0.8)$

$f_A > f_B$ for $\alpha \leq \tan^{-1}(0.8)\ f_A = f_B$]

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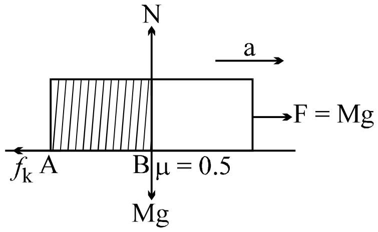
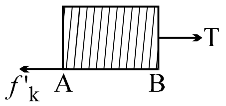
Q.21$_{nl/13}$ A rope of length L and mass M is being puled on a rough horizontal floor by a constant horizontal force F = Mg. The force is acting at one end of the rope in the same direction as the length of the rope. The coefficient of kinetic friction between rope and floor is 1/2. Then, the tension at the midpoint of the rope is

(A) Mg / 4 (B) 2Mg / 5 (C) Mg / 8 (D*) Mg / 2



[Sol. $a = \dfrac{F - \mu N}{M} = \dfrac{Mg - 0.5Mg}{M} = g/2$

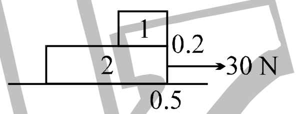


$T - \mu Mg/2 = Ma/2$
$T - Mg/4 = Mg/4$
$T = Mg/2$]

Q.22$_{nl/13}$ A plank of mass 2kg and length 1 m is placed on a horizontal floor. A small block of mass 1 kg is placed on top of the plank, at its right extreme end. The coefficient of friction between plank and floor is 0.5 and that between plank and block is 0.2. If a horizontal force = 30 N starts acting on the plank to the right, the time after which the block will fall off the plank is (g = 10 m/s^2)

(A*) (2/3) s (B) 1.5 s (C) 0.75 s (D) (4/3) s

[Sol. $a_{1/g} = 2 \text{ m/s}^2$



$a_{2/g} = \dfrac{30 - 2 - 15}{2} = \dfrac{13}{2} = 6.5 \text{ m/s}^2$

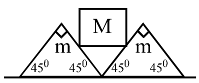
$a_{2/1} = 4.5$

$s_{2/1} = \dfrac{1}{2} a_{2/1} t^2$

$1 = \dfrac{1}{2} \times 4.5 \times t^2$

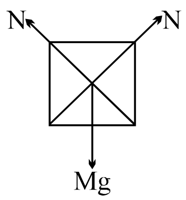
$t = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$ sec]

Q.23$_{nl/13}$ Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown. Assume no friction between the cube and the wedges, but a coefficient of static friction μ < 1 between the wedges and the floor. What is the largest M that can be balanced as shown without motion of the wedges?



(A) $\dfrac{m}{\sqrt{2}}$ (B) $\dfrac{\mu m}{\sqrt{2}}$ (C*) $\dfrac{2\mu m}{1-\mu}$ (D) All M will balance

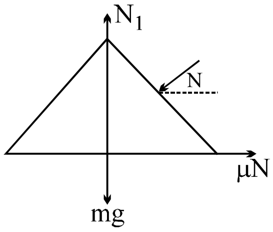
[Sol. $2N\cos 45 = Mg$ ...(1)



$\dfrac{N}{\sqrt{2}} + mg = N_1$ ...(2)

$\dfrac{N}{\sqrt{2}} = \mu N_1$ ...(3)

$$\frac{N}{\sqrt{2}} = \mu\left[\frac{N}{\sqrt{2}} + mg\right]$$

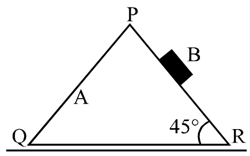


$$\Rightarrow N = \left(\frac{\sqrt{2}\mu}{1-\mu}\right)mg$$

$$\therefore \sqrt{2}\,N = \frac{2\mu}{1-\mu}\,mg = Mg$$

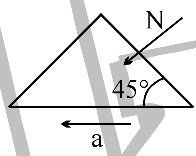
$$M = \frac{2\mu m}{1-\mu} \qquad]$$

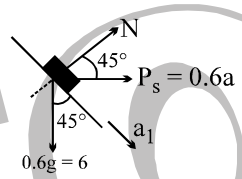
Q.24$_{nl/13}$ A particle B of mass 0.6 kg slides down the smooth face PR of a wedge A of mass 1.7 kg which can move freely on a smooth horizontal surface. The inclination of the face PR to the horizontal is 45°. Then:



(A*) the acceleration of A is 3g/20
(B*) the vertical component of the acceleration of B is 23 g/40
(C*) the horizontal component of the acceleration of B is 17 g/40
(D) none of these

[Sol. $\frac{N}{\sqrt{2}} = 1.7\ a$ ...(1)





$3\sqrt{2} + \frac{0.6a}{\sqrt{2}} = 0.6a_1$ ...(2)

$N + \frac{0.6a}{\sqrt{2}} = 3\sqrt{2}$ ...(3)

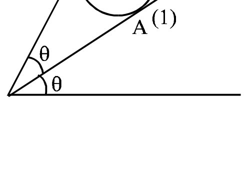
From eq (1) & (3)

$3 - 0.3a = 1.7\ a \quad \Rightarrow \quad a = 1.5\ m/s^2$

$3\sqrt{2} + \frac{0.9}{\sqrt{2}} = 0.6\ a_1 \quad \Rightarrow \quad a_1 = \frac{6.9}{0.6\sqrt{2}} = \frac{69}{6\sqrt{2}} = \frac{23}{2\sqrt{2}} \times \frac{g}{10} = \frac{23g}{20\sqrt{2}}$

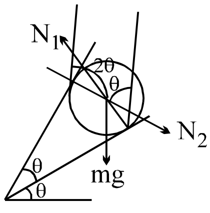
$a_{B/g} = \left(\frac{a_1}{\sqrt{2}} - a\right)\hat{i} - \frac{a_1}{\sqrt{2}}\,\hat{j} = \left(\frac{23}{40}g - \frac{1.5g}{10}\right)\hat{i} - \frac{23g}{40}\,\hat{j} = \frac{17g}{40}\,\hat{i} - \frac{23g}{40}\,\hat{j}$]

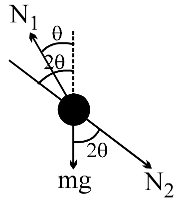
Q.25$_{nl/13}$ A sphere of mass m is kept between two inclined walls, as shown in the figure. If the coefficient of friction between each wall and the sphere is zero, then the ratio of normal reaction (N_1/N_2) offered by the walls 1 and 2 on the sphere will be



(A) tanθ
(B) tan2θ
(C*) 2cosθ
(D) cos2θ

[Sol.





$N_1\cos\theta = mg + N_2\cos 2\theta$ ...(1)
$N_1\sin\theta = N_2\sin 2\theta$ ...(2)

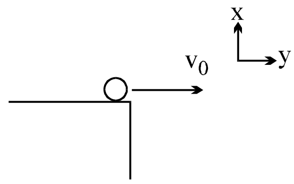
By eq (2)

$$\frac{N_1}{N_2} = 2\cos\theta \qquad]$$

Q.26$_{\text{circular/11/12/13}}$ A particle is projected horizontally from the top of a tower with a velocity v_0. If v be its velocity at any instant, then the radius of curvature of the path of the particle at the point (where the particle is at that instant) is directly proportional to:

(A) v^3 (B) v^2 (C) v (D) 1/v

[Sol. $\vec{v} = v_0\hat{i} - gt\hat{j}$ and $\vec{a} = -g\hat{j}$



$\therefore$ Component of $\vec{a} \perp$ to $\vec{v} = \vec{a} - \left(\frac{\vec{a}\cdot\vec{v}}{v^2}\right)\vec{v}$

i.e. $\vec{a}_\perp = \left(-\frac{v_0 g}{v_0^2 + g^2t^2}\right)(gt\hat{i} + v_0\hat{j})$

$\therefore$ $|\vec{a}_\perp| = \frac{v_0 g}{\sqrt{v_0^2 + gt^2}}$

Also, $r = \frac{|\vec{v}|^2}{|a_\perp|} = \frac{(v_0^2 + g^2t^2)^{3/2}}{v_0 g} = \frac{v^3}{v_0 g}$

$\therefore$ $r \propto v^3$

$\therefore$ Option (A) is correct]

Q.27$_{\text{wpe/11/12/13}}$There are two massless springs A and B of spring constant K_A and K_B respectively and $K_A > K_B$. If W_A and W_B be denoted as work done on A and work done on B respectively, then

(A) If they are compressed to same distance, $W_A > W_B$

(B) If they are compressed by same force (upto equilibrium state) $W_A < W_B$

(C) If they are compressed by same distance, $W_A = W_B$

(D) If they are compressed by same force (upto equilibrium state) $W_A > W_B$

[Sol. For same compression x_0 (say)

$$W_A = \frac{1}{2}k_A x_0^2 \ \ \& \ W_B = \frac{1}{2}k_B x_0^2$$

$\Rightarrow$ $W_A > W_B$ $[\therefore k_A > k_B]$

for same force at equilibrium force = F_0

$$x_A = \frac{F_0}{k_A}, x_B = \frac{F_0}{k_B}$$

$$\therefore \quad W_A = \frac{1}{2}k_A x_A^2 = \frac{{F_0}^2}{2k_A}$$

Similarly, $W_B = \frac{{F_0}^2}{2k_B}$

$\Rightarrow \quad W_B > W_A$

$\therefore$ (A) & (B) are correct options]

Q.28$_{wpe/13}$ A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is

(A) mgL (B) $\frac{mgL}{3}$ (C) $\frac{mgL}{9}$ (D*) $\frac{mgL}{18}$

[Sol. If hanging part of chain doesn't get any velocity then (D) is correct option. Also minimum work done is given by option (D) and its equal to change in gravitational potential energy of chain.

$\Delta W = \Delta PE \quad = \frac{M}{3} \times g \times \frac{L}{6}$ as CM moves by a distance L/6

$= \frac{MgL}{18}$

(D) is correct option]

Q.29$_{wpe/13}$ Power delivered to a body varies as $P = 3t^2$. Find out the change in kinetic energy of the body from t = 2 to t = 4 sec.

(A) 12 J (B*) 56 J (C) 24 J (D) 36 J

[Sol. Here power delivered is

$P = 3t^2$

If this power results into only kinetic energy change then

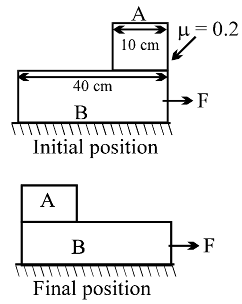
$$\Delta KE = \int_2^4 P\,dt = \int_2^4 3t^2\,dt = 3\left|\frac{t^3}{3}\right|_2^4 = (4^3 - 2^3)\text{ J} = 56\text{ J}$$

Power delivered will cause this maximum change in K.E.

(B) is correct option]

Q.30$_{wpe/13}$ A block 'A' of mass 45 kg is placed on a block 'B' of mass 123 kg. Now block 'B' is displaced by external agent by 50 cm horizontally towards right. During the same time block 'A' just reaches to the left end of block B. Initial & final position are shown in figure. Refer to the figure & find the workdone by frictional force on block A in ground frame during above time.

(A) – 18 Nm (B*) 18 Nm (C) 36 Nm (D) – 36 Nm



[Sol. Here blocks are moving w.r.t. each other, hence friction force = 0.2 × 45 × 10 = 90 N

Given block 'B" moves 50 cm

Also given that block A moves (40 – 10) cm back w.r.t. block 'B'

$\therefore$ Forward movement of block A in ground frame = 50 – 30 cm = 20 cm

$\therefore$ Work done by friction force = 90 × 0.2 J = 18 J

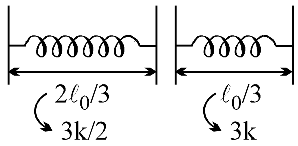
Work done is positive

$\therefore$ Option (B) is correct]

Q.31$_{wpe/13}$ A spring of force constant k is cut in two part at its one third length. when both the parts are stretched by same amount. The work done in the two parts, will be

(A) equal in both
(B) greater for the longer part
(C*) greater for the shorter part
(D) data insufficient.

[Sol. When a spring is cut into two parts each part has spring constant more than that of original spring. If k = spring constant & ℓ_0 = natural length, then for cut parts



If they are stretched by same amount then work done in shorter part will be double than that in the case of longer part.

$\therefore$ Option (C) is correct]

Q.32$_{wpe/13}$ The horsepower of a pump of efficiency 80%, which sucks up water from 10 m below ground and ejects it through a pipe opening at ground level of area 2 cm^2 with a velocity of 10 m/s, is about

(A) 1.0 hp (B*) 0.5 hp (C) 0.75 hp (D) 4.5 hp

[Sol. Here,

area = 2 cm^2 = 2×10^{-4} m^2

velocity = 10 m/s

$\therefore$ Volume flow rate = 2×10^{-3} m^3s^{-1} = vρgh

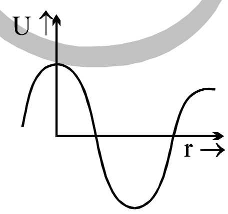
$\therefore$ Energy required per second = $100 \times 10^3 \times 2 \times 10^{-3}$ J = 2×100 J = 200 J

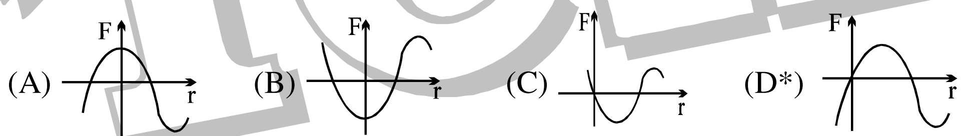
$\because$ Efficiency is 80%

$\therefore$ Power of pump = 250 W

Hence (B) is correct option]

Q.33$_{wpe/13}$ Potential energy and position for a conservative force are plotted in graph shown. Then force position graph can be





[Sol. Here by the graph we can say that

$U = U_0 \cos r$

$\therefore$ $F = -\frac{dU}{dr} = -U_0(-\sin r)$

$F = U_0 \sin r$

Hence correct option is (D)]

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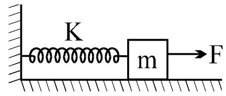
Q.34$_{wpe/13}$ A constant force produces maximum velocity V on the block connected to the spring of force constant K as shown in the fig. When the force constant of spring becomes 4K, the maximum velocity of the block is



(A) $\frac{V}{4}$ (B) 2V (C*) $\frac{V}{2}$ (D) V

[Sol. Block will gain maximum velocity at the point of equilibrium

In first case equilibrium elongation = $\frac{F}{k}$

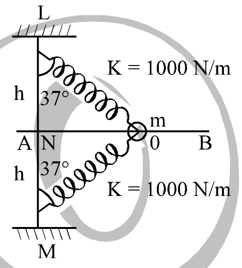
$$\therefore \quad F \cdot \frac{F}{k} - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mV^2 \quad \Rightarrow \quad V = \sqrt{\frac{F^2}{mk}}$$

In second case equilibrium elongation = $\frac{F}{4k}$

$$F \cdot \frac{F}{4k} - \frac{1}{2} \times 4k\left(\frac{F}{4k}\right)^2 = \frac{1}{2}mV'^2 \quad \Rightarrow \quad V' = \sqrt{\frac{F^2}{4mk}} = \frac{V}{2}$$

$\therefore$ (C) is correct option]

Q.35$_{wpe/13}$ A bead of mass 5kg is free to slide on the horizontal rod AB. They are connected to two identical springs of natural length h ms. as shown. If initially bead was at O & M is vertically below L then, velocity of bead at point N will be



(A*) 5h m/s (B) 40h/3 m/s
(C) 8h m/s (D) none of these

[Sol. Natural length of each spring is h

$\therefore$ elongation in each spring = $\frac{h}{\cos 37°} - h = \frac{h}{4}$

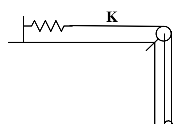
& applying work-energy theorem

$$\frac{1}{2}mv^2 = 2 \times \frac{1}{2}k\left(\frac{h}{4}\right)^2$$

v = 5h m/s

$\therefore$ Option (A) is correct]

Q.36$_{wpe/13}$ Block A in the figure is released from rest when the extension in the spring is x_0. The maximum downwards displacement of the block is



(A*) $\frac{Mg}{2K} - x_0$ (B) $\frac{Mg}{2K} + x_0$ (C) $\frac{2Mg}{K} - x_0$ (D) $\frac{2Mg}{K} + x_0$

[Sol. Let the block move 'x' downward then elongation in spring is '2x'

$$\therefore \quad \frac{1}{2}k(x_0 + 2x)^2 - \frac{1}{2}k\,x_0^2 = Mgx$$

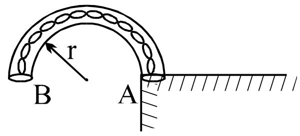
$$\Rightarrow k\,x_0^2 + 4kxx_0 + 4kx^2 - k\,x_0^2 = 2Mgx$$

$$\because \quad x \neq 0 \quad \Rightarrow \quad x_0 + x = \frac{Mg}{2k}$$

$\therefore \quad x = \frac{Mg}{2k} - x_0$

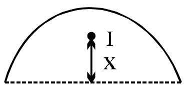
$\therefore$ Option (A) is correct]

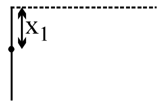
Q.37$_{wpe/13}$ A smooth semicircular tube AB of radius r is fixed in a vertical plane and contains a heavy flexible chain of length πr and weight $W = \pi r$ as shown. Assuming a slight disturbance to start the chain in motion, the velocity v with which it will emerge from the open end B of the tube is



(A) $\frac{4gr}{\pi}$ (B) $\frac{2gr}{\pi}$ (C) $\sqrt{2gr\left(\frac{2}{\pi}+\pi\right)}$ (D*) $\sqrt{2gr\left(\frac{2}{\pi}+\frac{\pi}{2}\right)}$

[Sol. Initial CM position Final CM position





$x = \frac{2r}{\pi}$ $x_1 = \frac{\pi r}{2}$

$\therefore \quad \Delta h$ for CM $= x + x_1$

$\Delta PE = \Delta KE \quad \Rightarrow \quad W\Delta h = \frac{1}{2}\frac{W}{g}U^2$

$U^2 = 2gr\left(\frac{2}{\pi}+\frac{\pi}{2}\right)$

$U = \sqrt{2gr\left(\frac{2}{\pi}+\frac{\pi}{2}\right)}$

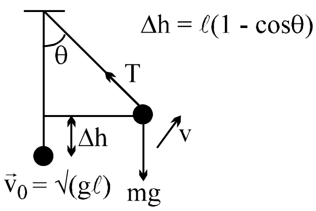
$\therefore$ Option (D) is correct]

Q.38$_{wpe/13}$ A heavy particle hanging from a string of length l is projected horizontally with speed $\sqrt{gl}$. The speed of the particle at the point where the tension in the string equals weight of the particle is:

(A) $\sqrt{2gl}$ (B) $\sqrt{3gl}$ (C) $\sqrt{gl/2}$ (D*) $\sqrt{gl/3}$

[Sol. Speed at bottom $= \sqrt{g\ell} < \sqrt{2g\ell}$

$mg\ell(1-\cos\theta) = \frac{1}{2}mg\ell - \frac{1}{2}mv^2 \quad ...(1)$



Also, $T - mg\cos\theta = \frac{mv^2}{\ell}$

But $T = mg$

$\therefore \quad \frac{mv^2}{\ell} = mg - mg\cos\theta$

i.e. $\frac{1}{2}mv^2 = \frac{mg\ell}{2}(1-\cos\theta)$

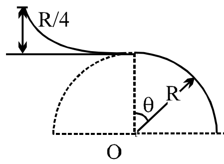
$\therefore$ eq^n (1) $\Rightarrow \quad mg\ell(1-\cos\theta) = \frac{1}{2}mg\ell - \frac{1}{2}mg\ell(1-\cos\theta)$

$$1 - \cos\theta = \frac{1}{3} \quad \Rightarrow \quad \cos\theta = \frac{2}{3}$$

$\therefore \quad v = \sqrt{g\ell/3}$

$\therefore$ Option (D) is correct]

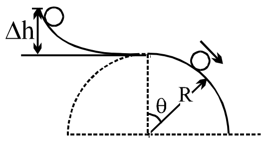
Q.39$_{wpe/13}$ A skier plans to ski a smooth fixed hemisphere of radius R. He starts from rest from a curved smooth surface of height (R/4). The angle θ at which he leaves the hemisphere is



(A) $\cos^{-1}(2/3)$ (B) $\cos^{-1}(5/\sqrt{3})$

(C*) $\cos^{-1}(5/6)$ (D) $\cos^{-1}(5/2\sqrt{3})$

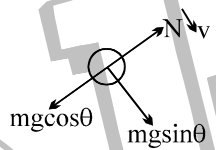
[Sol. $\Delta h = \frac{R}{4} + R(1 - \cos\theta)$



$$\frac{1}{2}mv^2 = mg\Delta h = \frac{mgR}{4}\{1 + 4(1 - \cos\theta)\}$$

$$\therefore \quad \frac{mv^2}{R} = \frac{mg}{2}(5 - 4\cos\theta)$$

$$mg\cos\theta - N = \frac{mv^2}{R}$$

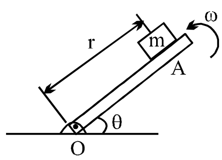


$$mg\cos\theta = \frac{mg}{2}(5 - 4\cos\theta)$$

$\cos\theta = 5/6$

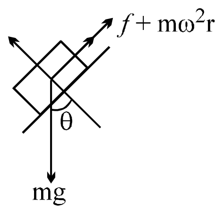
$\therefore$ Option (C) is correct]

Q.40$_{wpe/13}$ The inclined plane OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity ω = 3 rad/sec. As it passes the position θ = 0, a small mass m = 1 kg is placed upon it at a radial distance r = 0.5 m. If the mass is observed to be at rest with respect to inclined plane. The value of static friction force at θ = 37° between the mass & the incline plane.



(A*) 1.5 N (B) 3.5 N (C) 2.4 N (D) none

[Sol. Drawing the FBD in rotating frame we get



As the block is at rest, hence

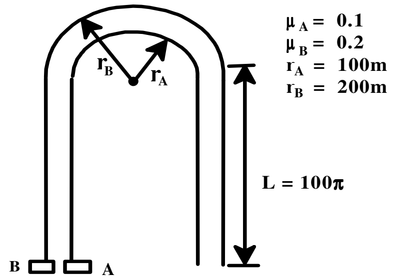
$mg\sin\theta = f + m\omega^2 r$

$\therefore \quad f = mg\sin\theta - m\omega^2 r = 1 \times 10 \times (3/5) - 1 \times 9 \times 0.5 = 6 - 4.5 = 1.5$

Therefore, force of friction (static) = 1.5 N

$\therefore$ (A) option is correct]

Q.41$_{wpe/13}$ Two cars A and B start racing at the same time on a flat race track which consists of two straight sections each of length 100π and one circular section as in Fig. The rule of the race is that each car must travel at constant speed at all times without ever skidding



(A) car A completes its journey before car B
(B) both cars complete their journey in same time
(C) velocity of car A is greater than that of car B
(D*) car B completes its journey before car A.

[Sol. $v \le \sqrt{\mu r g}$

$v_A \le \sqrt{0.1 \times 100 \times 10} = 10$ m/s

$v_B \le \sqrt{0.2 \times 200 \times 10} = 20$ m/s

$$t_A = \frac{200\pi + \pi(100)\,m}{10\,m/s} = 30\,\pi \text{ sec.}$$

$$t_B = \frac{200\pi + \pi(200)}{20\,m/s} = 20\,\pi \text{ sec.}$$]

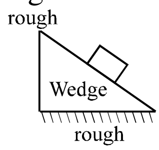
Q.42$_{wpe/13}$ A horizontal curve on a racing track is banked at a 45° angle. When a vehicle goes around this curve at the curve's safe speed (no friction needed to stay on the track), what is its centripetal acceleration?
(A*) g (B) 2g (C) 0.5g (D) none

[Sol. $\tan\theta = \frac{v^2}{Rg}$

$$\tan 45° = \frac{\frac{v^2}{R}}{g}$$

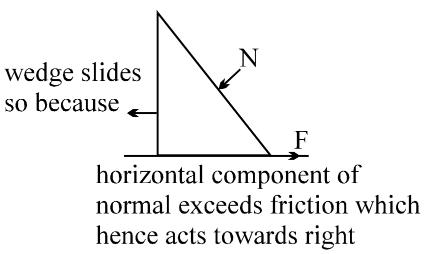
$$\frac{v^2}{R} = a_c = g$$]

Q.43$_{mom/11/12/13}$ When a block is placed on a wedge as shown in figure, the block starts sliding down and the wedge also start sliding on ground. All surfaces are rough. The centre of mass of (wedge + block) system will move

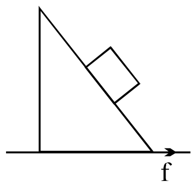


(A) leftward and downward (B*) rightward and downward
(C) leftward and upward (D) only downward

[Sol. (B) Wedge



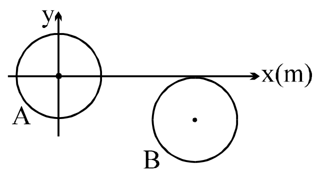
System



external force are gravity friction towards right so com shifts right + downward]

Question No. 44 to 46 (3 questions)

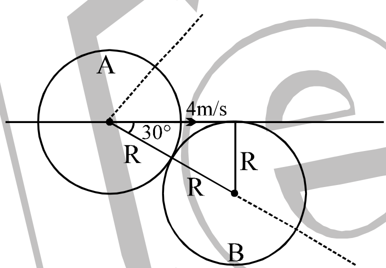
Two smooth balls A and B, each of mass m and radius R, have their centres at (0,0,R) and at (5R,–R,R) respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic.

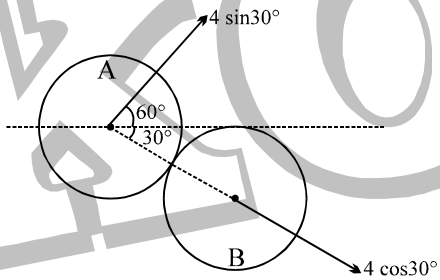


Q.44$_{mom/11/12/13}$ Velocity of the ball A just after the collision is

(A*) $(i+\sqrt{3}j)$ m/s (B) $(i-\sqrt{3}j)$ m/s (C) $(2i+\sqrt{3}j)$ m/s (D) $(2i+2j)$ m/s

[Sol. (A) Before collision After collision





$v_A = 4\sin 30°\,[\cos 60\,\hat{i} + \sin 60\,\hat{j}]$

$v_A = \hat{i} + \sqrt{3}\,\hat{j}$]

Q.45$_{mom/11/12/13}$ Impulse of the force exerted by A on B during the collision, is equal to

(A) $(\sqrt{3}mi + 3mj)$ kg-m/s (B) $(\frac{\sqrt{3}}{2}mi - \sqrt{3}mj)$ kg-m/s

(C*) $(3mi - \sqrt{3}mj)$ kg-m/s (D) $(2\sqrt{3}mi + 3mj)$ kg-m/s

[Sol. $\overrightarrow{J_{A\,on\,B}} = m\overrightarrow{V_{B_f}} - \overrightarrow{V_{B_i}}$

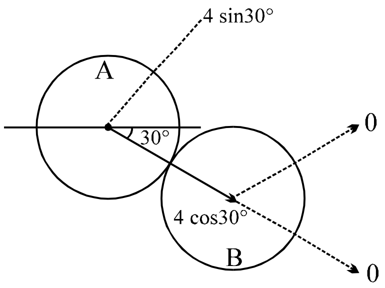
$= m\,[4\cos 30°(\cos 30°\hat{i} - \sin 30°\hat{j}) - 0]$

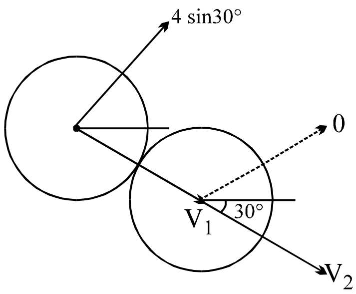
$= (3mi - \sqrt{3}mj)$ kg-m/s]

Q.46$_{mom/11/12/13}$ Coefficient of restitution during the collision is changed to 1/2, keeping all other parameters unchanged. What is the velocity of the ball B after the collision?

(A) $\frac{1}{2}(3\sqrt{3}i + 9j)$ m/s (B*) $\frac{1}{4}(9i - 3\sqrt{3}j)$ m/s (C) $(6i + 3\sqrt{3}j)$ m/s (D) $(6i - 3\sqrt{3})$ m/s

[Sol. Before collision After collision





(1) $\frac{1}{2} = \frac{-(V_2 - V_1)}{(0 - 4\cos 30°)}$

$V_2 - V_1 = \sqrt{3}$

(2) $m\frac{4\sqrt{3}}{2} = mV_1 + mV_2$

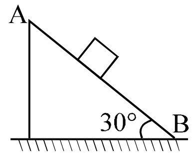
$V_2 + V_1 = 2\sqrt{3}$

$V_2 = \frac{3\sqrt{3}}{2}$ m/s

$\vec{V}_2 = \frac{3\sqrt{3}}{2}\ [\cos 30°\,\hat{i} + \sin 30°\,(-\hat{j})]$

$= \frac{9}{4}\hat{i} - \frac{3\sqrt{4}}{4}\hat{j}$]

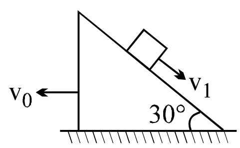
Q.47$_{mom11/12/13}$ A particle of mass m = 0.1 kg is released from rest from a point A of a wedge of mass M = 2.4 kg free to slide on a frictionless horizontal plane. The particle slides down the smooth face AB of the wedge. When the velocity of the wedge is 0.2 m/s the velocity of the particle in m/s relative to the wedge is:



(A) 4.8 (B*) $\frac{10}{\sqrt{3}}$ (C) 7.5 (D) 10

[Sol. (1) $O = 0.1\,(v_1 \cos 30° - v_0) - 2.4\, v_0$

$v_1 \cos 30° = 25\, v_0$



(2) $v_1 = \frac{25(0.2)}{\sqrt{3}/2} = \frac{5}{\sqrt{3}/2} = \frac{10}{\sqrt{3}}$ m/s]

Q.48$_{mom/12/13}$ A bullet of mass 0.01 kg and travelling at a speed of 500 m/s strike a block of mass of 2 kg which is suspended by a string of length 5 m. The centre of gravity of the block is found to rise a vertical distance of 0.1 m. What is the speed of the bullet after it merge from the block:

(A) 780 m/s (B*) 220 m/s (C) 1.4 m/s (D) 7.8 m/s

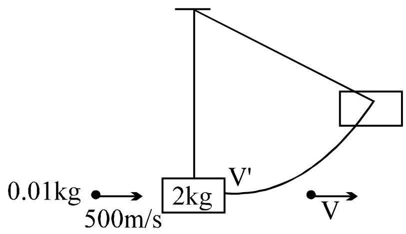
[Sol. For block

$V'^2 = 2gh$

$V'^2 = 2\times10\times0.1$

$V' = \sqrt{2}$ m/s

Just before and after bullet strikes, momentum conserved



$0.01 \times 500 = 2V' + 0.01\ V$

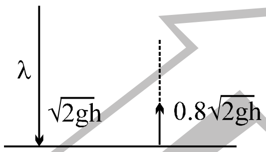
$5 - 2\sqrt{2} = 0.01\ V$

$V = \dfrac{5-2.828}{0.01} = 217.2$ m/s]

Q.49$_{mom/13}$ A ball is dropped from a height h. As it bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly

(A) 0.80 h (B) 0.75 h (C*) 0.64 h (D) 0.50 h

[Sol.



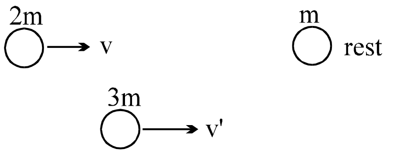
$h' = \dfrac{(0.8\sqrt{2gh})}{2g}$

$= 0.64$ h]

Q.50$_{mom/13}$ In a one-dimensional collision, a particle of mass 2m collides with a particle of mass m at rest. If the particles stick together after the collision, what fraction of the initial kinetic energy is lost in the collision?

(A) $\dfrac{1}{4}$ (B*) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) none

[Sol.



$2mv + 0 = 3mv' \Rightarrow v' = \dfrac{2}{3}v$

$\dfrac{\Delta K}{Ki} = 1 - \dfrac{K_f}{K_i} = 1 - \dfrac{\frac{1}{2}3m\left(\frac{2}{3}v\right)^2}{\frac{1}{2}2mv^2}$

$= 1 - \dfrac{3}{2}\times\dfrac{4}{5} \qquad \Rightarrow \dfrac{1}{3}$]

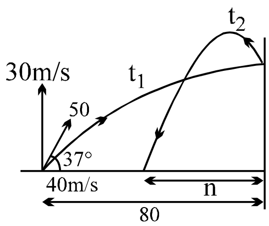
Q.51$_{mom/13}$ A particle is projected from ground towards a vertical wall 80m away at an angle of 37° with horizontal with initial velocity of 50m/s. After its collision with wall & then once with ground find at what distance from wall will it strike the ground again if coefficient of restitution for both collisions is equal to 1/2.

(A) 70 m (B) 120 m (C*) 140 m (D) none

[Sol. **After first collision**

$V_y = 30 - gt = 30 - 10\left(\frac{80}{40}\right) = (10\text{m/s})\,\hat{j}$

$V_x = -\frac{1}{2}(40) = -20\hat{i}$



$t_1 = \frac{80}{40} = 2$ sec

$t_2 = T - t_1 = \frac{2\times30}{10} - 2 = 4$ sec

Before second collision

$V_x = -\ 20\hat{i}$

$x = 20 \times 4 = -\ 80$ m

$V_y = 10 - 10\,(t_2) = 10 - 10(4) = -30\,\hat{j}$

After second collision

$V_x = -\ 20\hat{i}$

$V_y = +15\,\hat{j}$

Range $= \frac{2\times20\times15}{10}$

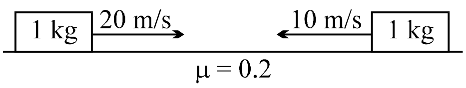
= 60 m

Net:→ 60 m + 80 m

= 140 m]

Question No. 52 & 53 (2 Questions)

Two blocks (from very far apart) are approaching towards each other with velocities as shown in figure. The coefficient of friction for both the blocks is $\mu = 0.2$



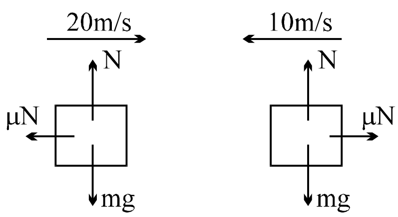
Q.52$_{mom/13}$ Linear momentum of the system is

(A) conserved all the time (B) never conserved

(C*) is conserved upto 5 seconds (D) none of these

[Sol. $F_{net} = 0$ till = 5 second

$V_{cm} = \frac{20-10}{2} = 5$ m/s



]

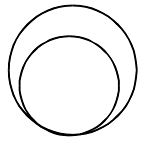
Q.53$_{mom/13}$ How much distance will centre of mass travel before coming permanently to rest
(A) 25 m (B*) 37.5 m (C) 42.5 m (D) 50 m

[Sol. $(m_1 + m_2)\ \overrightarrow{\Delta X}_{cm} = m_1 \overrightarrow{X_1} + m_2 \overrightarrow{X_2}$ $\overrightarrow{X_1} = -10(5) + \frac{1}{2} \times 2(5)^2 = 25$

$2\ (\overrightarrow{\Delta X}_{cm}) = -25 + 100 = -75$ $\overrightarrow{X_2} = 20\ (10) - \frac{1}{2} \times 2 \times (10)^2 = 100$

$|\overrightarrow{\Delta X}_{cm}| = 37.5$ m]

Q.54$_{mom/13}$ From a thin circular disc of radius R, a circular hole of radius 4R/5 is cut as shown. The distance of the centre of mass of remaining disc, from the centre of the original disc is



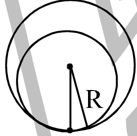
(A) 15R/40 (B) R/3
(C) R/4 (D*) 16R/45

[Sol. $\sigma = \frac{m}{\pi R^2}$

$m' = \frac{m}{\pi R^2} \times \pi \left(\frac{4R}{5}\right)^2$

$m' = \frac{16}{25} m$

$X_{cm} = \frac{0 \times 0 - \left(\frac{m16}{25}\right)\left(\frac{R}{5}\right)}{m - \frac{16m}{25}}$



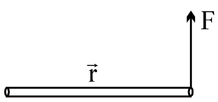
$R - \frac{4R}{5} = \frac{R}{5}$

$X_{cm} = \frac{-mR\left(\frac{16}{125}\right)}{\frac{9m}{25}} = -R\ \frac{16}{125} \times 25$

$\Rightarrow X_{cm} = \frac{-16R}{45}$]

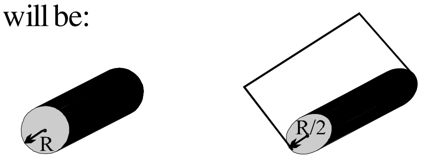
Q.55$_{rot/11/12/13}$ The density of a rod gradually decreases from one end to the other. It is provided at an end so that it can move about a vertical axis through the pivot. A horizontal force F is applied on the free end in a direction perpendicular to the rod. The quantities, that do not depend on which end of the rod is pivoted, are
(A) angular acceleration (B) angular velocity when the rod completes on rotation
(C) angular momentum when the rod completes one rotation
(D*) torque of the applied force

[Sol. $\vec{\tau} = \vec{r} \times \vec{F} = r\ F \sin 90 = rF$



$\tau = I\ \alpha$
I will a vary]

Q.56$_{rot/12/13}$ A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. The horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2 will be:



(A*) $v = \sqrt{\dfrac{14gR}{3}}$ (B) $v = \sqrt{\dfrac{2gR}{3}}$ (C) $\sqrt{2gR}$ (D) $\sqrt{5gR}$

[Sol. $m' = \sigma\left(\pi\left(\dfrac{R}{2}\right)^2\right)$

$m' = \sigma\dfrac{\pi R^2}{4}$

$m' = \dfrac{m}{4}$

C.O.E.

$mgR = \left(\dfrac{m}{4}\right)g\left(\dfrac{R}{2}\right) + \dfrac{1}{2}I\,w^2 + \dfrac{1}{2}m'\,v^2$

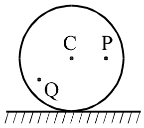
$mgR = \dfrac{mgR}{8} + \dfrac{1}{2}\left(\dfrac{m}{4}\right)\dfrac{1}{2}\left(\dfrac{R}{2}\right)^2 w^2\left(\dfrac{v}{(R/2)}\right)^2 + \dfrac{1}{2}\left(\dfrac{m}{4}\right)v^2$

$mgR = \dfrac{mgR}{8} + \dfrac{1}{16}mv^2 + \dfrac{1m}{8}v^2$

$\dfrac{7mgR}{8} = \dfrac{1+2}{16}mv^2$

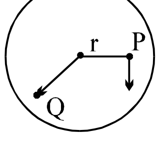
$v = \sqrt{\dfrac{14gR}{3}}$]

Q.57$_{rot/12/13}$ A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is:



(A) $v_Q > v_C > v_P$ (B) $v_P > v_C > v_Q$ (C) $v_P = v_C = v_p/2$ (D*) $v_P > v_C > v_Q$

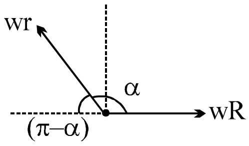
[Sol. $v_p^2 = (wR)^2 + (w\,r)^2 = w\sqrt{R^2 + r^2}$



$v_Q^2 = (wR)^2 + (wr)^2 + 2(wR)(wr)\cos(\pi - \alpha)$

$v_Q^2 = w^2\sqrt{R^2 + r^2 - 2Rr\,w\,r\alpha}$

since $\cos\alpha < 1$ Let $\alpha = 45$



$v_Q = w\sqrt{R^2 + r^2 - Rr\sqrt{2}}$

$v_C = wR$

$v_P > v_C > v_Q$]

Q.58$_{\text{heat/11/12/13}}$ The liquid of a liquid-thermometer should have the following properties:

(A) Large value of specific heat & low value of coefficient of the small expansion

(B*) Small value of specific heat & large value of coefficient of the small expansion

(C) Large value of boiling point and low value of freezing point

(D) Low value of boiling point and large value of freezing point

[Sol. Small value of specific heat and large value of coefficient of the small expansion.]

Q.59$_{\text{thermal/11/12/13}}$ A composite bar of length $L = L_1 + L_2$ is made up from a rod of material 1 and of length L_1 attached to a rod of material 2 and of length L_2 as shown. If α_1 and α_2 are their respective coefficients of linear expansion, then equivalent coefficient of linear expansion for the composite rod is:

1	2

(A) $\dfrac{\alpha_1 L_2 + \alpha_2 L_1}{L}$ (B) $\dfrac{\alpha_1 L_2 + \alpha_2 L_2}{L}$ (C*) $\dfrac{\alpha_1 L_1 + \alpha_2 L_2}{L}$ (D) $\dfrac{\alpha_1\alpha_2(L_1^2 + L_2)}{(\alpha_1 L_1 + \alpha_2 L_2)}$

[Sol. $\alpha_{eq}(L_1 + L_2)\,\Delta T = L_1\,\alpha_1\,\Delta T + L_2\,\alpha_2\,\Delta T$

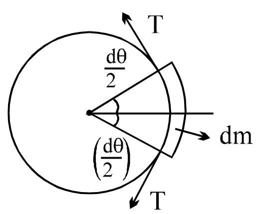
$\alpha_{eq} = \dfrac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$]

Q.60$_{\text{heat/13}}$ If the maximum tension the ring can withstand is F_{max} and its linear mass density is d. The maximum permissible linear velocity of a rotating thin lead ring (axis of rotation is the axis of the ring) is

(A) $\sqrt{\dfrac{F_{Max}}{2d}}$ (B) $\sqrt{\dfrac{2F_{Max}}{d}}$ (C*) $\sqrt{\dfrac{F_{Max}}{d}}$ (D) $\sqrt{\dfrac{F_{Max}}{3d}}$

[Sol. $T\cos\left(\dfrac{d\theta}{2}\right) = T\cos\left(\dfrac{d\theta}{2}\right)$

$2T\sin\left(\dfrac{d\theta}{2}\right) = (dm)\,w^2 r$



$2T\left(\dfrac{d\theta}{2}\right) = \left(\dfrac{m}{L}\right) r\,d\theta \times r\,w^2$

$T = \dfrac{mw^2r^2}{2\pi r}$

$F_{max} = T = \left(\dfrac{m}{L}\right)v^2$

$T = \dfrac{m}{L}v^2$

$\sqrt{\dfrac{F_{max}}{d}} = v$]

Q.61$_{heat/13}$ A vessel containing a liquid is heated with its contents. The pressure at the bottom of vessel due to the liquid should. (no evaporation take place)

(A) increase
(B*) decrease
(C) increases if $\gamma_{liq} > 3\,\alpha_{vessel}$
(D) decreases if $\gamma_{liq} > 3\,\alpha_{vessel}$

[Sol. $P = \frac{F}{A}$ $\quad P \propto \frac{1}{A}$

$\frac{dP}{P}$ = decrease $= -\frac{dA}{A}$]

Q.62$_{heat/13}$ 10 gm of ice at 0°C is mixed with '*m*' gm of water at 50°C. What is minimum value of m so that ice melts completely. (L_f = 80 cal/gm and s_w = 1 cal/gm-°C)

(A) 32 gm
(B) 20 gm
(C) 40 gm
(D*) 16 gm

[Sol. $Q_{gain} = Q_{cost}$

$10 \times 80 = 50 \times m \times 1$

$800 = 50 + m$

$m = \frac{800}{50} = 16$ gm]

Q.63$_{thermo/12/13}$ A closed vessel contains a mixture of two diatomic gases A and B. Molar mass of A is 16 times and that of B and mass of gas A, contained in the vessel is 2 times that of B.

(A*) Average kinetic energy per molecule of gas A is equal to that of gas B
(B*) Root mean square value of translational velocity of gas B is four times that of A
(C*) Pressure exerted by gas B is eight times of that exerted by gas A
(D*) Number of molecules of gas B in the cylinder is eight times that of gas A

[Sol. $\frac{5}{2}KT$ $\quad$ avg. KE is same $\rightarrow$ A

$V_{rms\,A} = \sqrt{\frac{3RT}{M}}$, $V_{rms\,B} = \sqrt{\frac{3RT}{(M/16)}} = 4V_{rms\,A} \rightarrow B$

$n_A = \frac{2m}{M}$, $n_B = \frac{2m}{(M/16)} = 16m/M$

$P_B = \left(\frac{n_B}{n_A + n_B}\right)P_0$, $P_A = \left(\frac{n_A}{n_A + n_B}\right)P_0$

$\Bigg\} \rightarrow C, D$]

Q.64$_{thermo/12/13}$ A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments whose initial parameters are given. The partition is a conducting wall which can move freely without friction. Which of the following statements is/are correct, with reference to the final equilibrium position?

(A*) The pressure in the two compartments are equal
(B*) Volume of compartment I is 3V/5
(C*) Volume of compartment II is 12V/5
(D*) Final pressure in compartment I is 5P/3

P, V, T I	2P, 2V, T II

[Sol. $\frac{PV}{T} = \frac{P_1V_1}{T}$ Temp. will not change as internal energy of the system will remain unchanged

For 2nd compartment

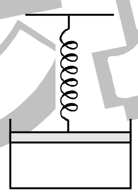
$\frac{(2P)2V}{T} = \frac{P_1(3V - V_1)}{T}$ $\Rightarrow$ $4PV = \frac{PV}{V_1}(3V - V_1)$

$\Rightarrow$ $4V_1 = 3V - V_1$

$\Rightarrow$ $V_1 = \frac{3V}{5}$, $V_2 = \frac{12V}{5}$

$P_1 = \frac{5P}{3}$]

Q.65$_{thermo/12/13}$ One mole of an ideal gas is kept enclosed under a light piston (area = 10^{-2} m^2) connected by a compressed spring (spring constant 100 N/m). The volume of gas is 0.83 m^3 and its temperature is 100 K. The gas is heated so that it compresses the spring further by 0.1 m. The work done by the gas in the process is (Take R = 8.3 J/mole and suppose there is no atmosphere):



(A) 3 J (B) 6 J (C) 9 J (D*) 1.5 J

[Sol. $kx_0 = PA$ [x_0 is initial compression]

$100x_0 = \left(\frac{nRT}{V}\right) \times 10^{-2}$

$x_0 = \frac{1}{100} \times \frac{1\times 8.3\times 100}{0.83} \times \frac{1}{100} = 0.1$

$x_1 = 0.2$ (total compression)

$W_{gas} = \frac{1}{2} \times 100 \times (0.2)^2 - \frac{1}{2} \times 100 \times (0.1)^2 = \frac{100}{2} \times \frac{1}{100}[4 - 1] = 1.5$]

Q.66$_{thermo/12/13}$ Number of collisions of molecules of a gas on the wall of a container per m^2 will:

(A) Increase if temperature and volume both are doubled
(B) Increase if temperature and volume both are halved
(C) Increase if pressure and temperature both are doubled

(D) Increase if pressure and temperature both are halved

[Sol. $N(2mV) = P$

$$N \propto \frac{P}{\sqrt{T}} = \sqrt{\frac{T}{V}} =$$]

Q.67$_{go/12/13}$ The image produced by a concave mirror is one quarter the size of object. If the object is moved 5 cm closer to the mirror, the image will only be half the size of the object. The focal length of mirror is

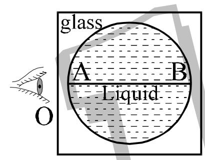
(A) f = 5.0 cm (B*) f = 2.5 cm (C) f = 7.5 cm (D) f = 10 cm

[Sol. $$\frac{1}{4} = \frac{f}{x_0 - f}$$

$$\frac{1}{2} = \frac{f}{x_0 - 5 - f}$$

Solving $f = 2.5$]

Q.68$_{go/12/13}$ The observer 'O' sees the distance AB as infinitely large. If refractive index of liquid is μ_1 and that of glass is μ_2, then μ_1/μ_2 is:



(A*) 2 (B) 1/2 (C) 4 (D) None of these

[Sol. B must be appearing at infinity

$$\frac{h_2}{\infty} - \frac{\mu_1}{-2R} = \frac{\mu_2 - \mu_1}{-R}$$

Solving $\frac{\mu_1}{\mu_2} = 2$]

Q.69$_{go/12/13}$ For a prism kept in air it is found that for an angle of incidence 60°, the angle of refraction 'A', angle of deviation 'δ' and angle of emergence 'e' become equal. Then the refractive index of the prism is

(A*) 1.73 (B) 1.15 (C) 1.5 (D) 1.33

[Sol. $\delta = e = A$

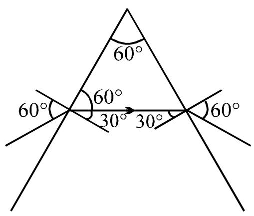
$\delta = i + e - A$

$\delta = i = 60°$

$e = 60°$

$A = 60°$

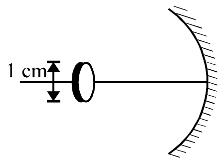
$1 \times \sin 60° = \mu \sin 30°$



$$\mu = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$$]

Question No. 70 to 72 (3 questions)

A concave mirror of radius of curvature 20 cm is shown in the figure. A circular disc of diameter 1 cm is placed on the principle axis of mirror with its plane perpendicular to the principal axis at a distance 15 cm from the pole of the mirror. The radius of disc increasing according to the law r = (0.5 + 0.1t) cm/sec.



Q.70$_{go/12/13}$ The image formed by the mirror will be in the shape of a:
(A*) circular disc
(B) elliptical disc with major axis horizontal
(C) elliptical disc with major axis vertical
(D) distorted disc

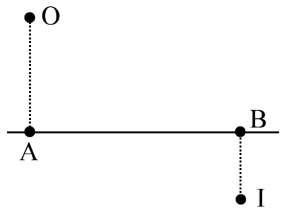
Q.71$_{go/12/13}$ In the **above question**, the area of image of the disc at t = 1 second is:
(A) 1.2 π cm^2 (B*) 1.44 π cm^2 (C) 1.52 π cm^2 (D) none of these

[Sol. $r_0 = 0.6$
$r_i = 0.6 \times 2$ (m = 2)
$= 1.2$
$A = \pi \times (1.2)^2 = 1.44\pi$ $\Rightarrow$ Option (B) is correct]

Q.72$_{go/12/13}$ What will be the rate at which the horizontal radius of image will be changing
(A*) 0.2 cm/sec increasing
(B) 0.2 cm/sec decreasing
(C) 0.4 cm/sec increasing
(D) 0.4 cm/sec decreasing

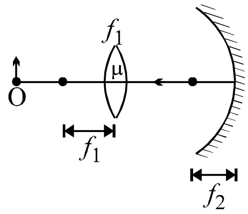
[Sol. $r_i = 2(0.5 + 0.1t) = 1 + 0.2t$
$\frac{dr_i}{dt} = 0.2$]

Q.73$_{go/12/13}$ A luminous point object is placed at O, whose image is formed at I as shown in figure. Line AB is the optical axis. Which of the following statement is/are correct?



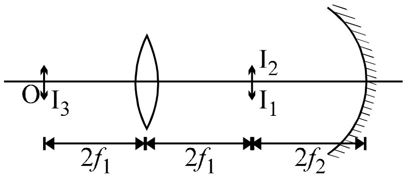
(A) If a lens is used to obtain the image, then it must be a diverging lens and its optical centre will be the intersection point of line AB and OI.
(B*) If a lens is used to obtain the image, then it must be a converging lens and its optical centre will be the intersection point of line AB and OI.
(C*) If a mirror is used to obtain the image then the mirror must be concave and object and image subtend equal angles at the pole of the mirror.
(D*) I is a real image.

Q.74$_{go/12/13}$ An object is placed infront of a converging lens at a distance equal to twice the focal length f_1 of the lens. On the other side of the lens is a concave mirror of focal length f_2 separated from the lens by a distance $2(f_1 + f_2)$. Light from the object passes rightward through the lens, reflects from the mirror, passes leftward through the lens and forms a final image of the object.



(A) The distance between the lens and the final image is equal to $2f_1$.
(B) The distance between the lens and the final image is equal to $2(f_1 + f_2)$
(C*) The final image is real, inverted and of same size as that of the object
(D) The final image is real, erect and of same size as that of the object

[Sol.



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Q.75$_{go/12/13}$ A parallel beam of light passes parallel to the axis and falls on one face of a thin convex lens of focal length f and after two internal reflections emerges from the second face and forms a real image. Find the distance of the image from the lens if μ is the refractive index of the lens.

(A*) f (μ–1) / 3μ–1 (B) (μ–1) / f (3 μ–1) (C) (3μ – 1)/f (μ – 1) (D) f (μ –1)

[Sol. Using formula]

ANSWER KEY

DIWALI ASSIGNMENT

Q.1	A	Q.2	C	Q.3	C	Q.4	D	Q.5	C	Q.6	A	Q.7	D
Q.8	C	Q.9	D	Q.10	B	Q.11	C	Q.12	A,D	Q.13	C	Q.14	A
Q.15	A	Q.16	A,D	Q.17	D	Q.18	C	Q.19	B	Q.20	B,C	Q.21	D
Q.22	A	Q.23	C	Q.24	A,B,C	Q.25	C	Q.26	A	Q.27	A,B	Q.28	D
Q.29	B	Q.30	B	Q.31	C	Q.32	B	Q.33	D	Q.34	C	Q.35	A
Q.36	A	Q.37	D	Q.38	D	Q.39	C	Q.40	A	Q.41	D	Q.42	A
Q.43	B	Q.44	A	Q.45	C	Q.46	B	Q.47	B	Q.48	B	Q.49	C
Q.50	B	Q.51	C	Q.52	C	Q.53	B	Q.54	D	Q.55	D	Q.56	A
Q.57	D	Q.58	B	Q.59	C	Q.60	C	Q.61	B	Q.62	D		
Q.63	A,B,C,D	Q.64	A,B,C,D	Q.65	D	Q.66		Q.67	B	Q.68	A		
Q.69	A	Q.70	A	Q.71	B	Q.72	A	Q.73	B,C,D	Q.74	C	Q.75	A